For office use only	Team Control Number	For office use only
T1 ________________	**23178**	F1 ________________
T2 ________________		F2 ________________
T3 ________________	Problem Chosen	F3 ________________
T4 ________________	**B**	F4 ________________

2013 Mathematical Contest in Modeling (MCM) Summary Sheet

(Attach a copy of this page to your solution paper.)

Summary

In this day and age, fresh water is the limiting constraint for development in much of the world. Inevitably, China is also facing the severe problem. Lots of provinces are keeping thirsty for a long time. Therefore, make a water strategy for China is extremely urgent.

Our modeling group has successfully built a model for water strategy to meet the projected water needs of 2025 for China. Our strategy contains several small models; each model can solve part of the problem for us.

First of all, our group analyzes the composition of freshwater—supply and demand. We use the statistic data of supply and demand to build our first model to address the **degree of freshwater shortage**. Using this model we divide the provinces of China to five big areas to simplify our discussion.

Secondly, we regard each area as a unit to predict the freshwater supply and demand of 2015. Because the demand includes four parts: water for living, industrial water, agricultural water and ecological water, we develop different models separately to predict every part to make the result more reasonable. We apply **Grey Model (GM)** and **Logistic population model** to the prediction of living water, and GM for other three aspects. In addition, we use **curve fitting model** to get the supply water for 2025. Then we test the predictions and find the matching are all good.

Thirdly, we analyze the difference between the supply and demand among areas to decide the storage and movement of freshwater. We format **Minimum Spanning Tree (MTS)** to choose the best movement routes and use **Fermat point** to choose a better position to build reservoirs to storage with the less cost. Furthermore, we analyze the cost of de-salinization and draw a conclusion that de-salinization is cheaper but only appropriate for the places closed to the sea.

Finally, we use our model to discuss the economic, physical, and environmental implications of our strategy and put forward the method to protect the water resources.

What' more, we provide a suggest paper to the government sincerely outlining our group's approach, its reasonability and costs wishing that the governmental leaders will take some more effective methods to protect the freshwater resources.

Contents

1. Introduction

In this day and age, fresh water is the limiting constraint for development in much of the world. Inevitably, China is facing the severe problem.

Across China, a lot of provinces are keep thirsty for a long time. The freshwater supply no longer meets the demand. What is worse, if we could not do something useful for this, China would come to a situation that no freshwater could be used just like petroleum which all countries are pursuing. Therefore, make a water strategy for China is extremely urgent.

Our modeling group has built a mathematical model for determining an effective, feasible, and cost-efficient water strategy to meet the projected water needs of 2025 for China. Our strategy contains several small models; each model can solve part of the problem for us.

First of all, we begin at analyzing the composition of freshwater—supply and demand. We use the statistic data of supply and demand which found in China Statistical Yearbook to build a model to address the degree of freshwater shortage. Using this model we divide the provinces of China to five big areas.

Secondly, we need to get the supply and demand of freshwater for each area in 2025 and use this to decide further steps to do. As we all know, the freshwater demand include four parts: water for living, industrial water, agricultural water and ecological water. So we use China's former year statistic data for each area to separately predict every part of the freshwater demand. Then we predict the supply water which contains surface and ground water for each area.

Thirdly, we analyze the difference between the supply and demand to decide the storage and movement of freshwater among areas. For one area, if the supply is greater than the demand, we should move out water to the area which doesn't have enough water supply. In addition, if one area needs water, we also analyze the cost of movement and de-salinization to choose a better method to solve the problem of freshwater shortage.

After analyzing the area, we analyze the situation of the country. If the whole country's supply doesn't meet the demand, we must get fresh water from the sea or the iceberg.

When we have done the model for the strategy, we put forward some ways to improve our model to make the result more precise and feasible.

Finally, we use our model to discuss the economic, physical, and environmental implications of our strategy and put forward the method to protect the water resources.

What' more, we provide a non-technical position paper to governmental leadership outlining our group's approach, its feasibility and costs, and why it is the "best water strategy choice."

2. Assumption

- We assume the statistical data getting from China Statistical Yearbook is genuine and believable since our model is based on the data analysis.
- The social environment keeps relatively stable in the following years, especially from 2013 to 2025.We don’t consider the objective mutation such as destructive ecological disaster.
- The weather effect to the model such as rainfall and the reclaimed water finally go into the surface and underground water resources.
- The fresh water won’t get lost in the storage and movement.
- China's family planning policy keeps carrying out and the rates of birth and death remain basically unchanged in these years.
- The costs of seawater desalination per ton of every area are the same.

3. The analysis of freshwater supply and demand

3.1 Basic Analysis of fresh water

China’s economic and cultural development and the living standards of its people have closed related to the fresh water. Fresh water contains two aspects: the supply and demand, so firstly our analysis and discussion focus on the contents of fresh water.

The freshwater supply includes surface water, underground water and the overlapping of the two; the freshwater demand is made up by four aspects: water for living, industrial water, agricultural water and ecological water (*please see the Figure1 below)*.

Our group separately predicts the factors of the supply and demand, then combine them we will get the freshwater supply and demand of 2025.

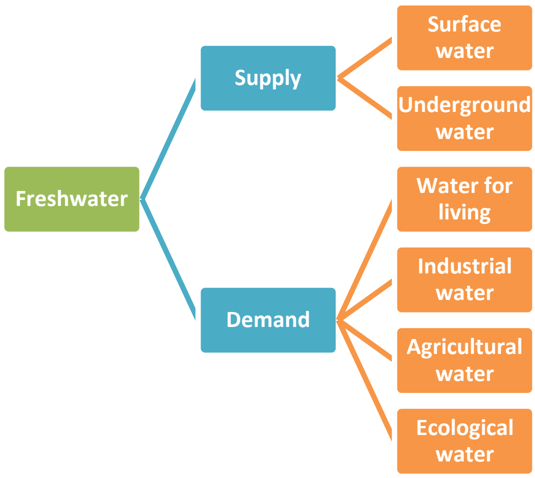

Figure1: The composition of fresh water

3.2 The model of water-shortage degree

To get a better and more concise model, we firstly build a model to evaluate the degree of freshwater shortage of each province. Using this model, we can divide all provinces into five areas. We use every area as a unit in the following discussion.

According to the statistic data (2004 to 2011) of every province's freshwater supply and demand from China Statistical Yearbook [1], we define the following parameters:

S_j^i: The freshwater's supply

D_j^i: The freshwater's demand of every province

P_j^i: The degree of water shortage of every province

$\overline{P_j}$: The average value of P_j^i

i=2004, 2005…, 2011, j=1, 2…, 31(the number of every province)

Then we build a water shortage degree function:

$$P_j^i = \frac{S_j^i - D_j^i}{S_j^i} \ ,$$

$$\overline{P_j} = \frac{\sum P_j^i}{8}$$

We have to say that because we couldn't find the data of Hong Kong, Macao and Taiwan, so our discussion doesn't include them.

Using this function to calculate, we can get $\overline{P_j}$ of every province. According to $\overline{P_j}$, we divide China into five areas.

We take Beijing as an example of analysis. The statistic data of Beijing from 2004 to 2011 is below: *(please see the Table1 below)*

Table1: The freshwater supply and demand of Beijing (2004-2011)

	2004	2005	2006	2007	2008	2009	2010	2011
Supply(100Million Cubic Meters)	18.4	21.3	23.2	22.1	23.8	34.2	21.8	23.1
Demand(100Million Cubic Meters)	35.0	34.6	34.5	34.3	34.8	35.1	35.5	35.2

Table2: The degree of water shortage of Beijing (2004-2011)

	2004	2005	2006	2007	2008	2009	2010	2011
S-D	-16.604	-13.210	-11.320	-12.227	-10.996	-0.880	-13.659	-12.116
P	-0.903	-0.619	-0.488	-0.554	-0.462	-0.026	-0.625	-0.525

Then we get P_j^i of Beijing. *(Please see the Table2 above)*

$$\overline{P_j}=\frac{\sum P_j^i}{8} \approx -0.525$$

Similarly, we have applied the analysis to the other 30 provinces we have chosen and get the five levels of water shortage degree as follows. *(Please see the Table3 below.)* The degree increases with the level increases.

Table3: Five levels of province and areas of the nation

Level one

Province	$\overline{P_J}$
Tibet	0.993
Qinghai	0.955
Yunnan	0.922
Sichuan	0.910
Guizhou	0.896
Chongqing	0.851
Hainan	0.845
Jiangxi	0.840

Level Two

Province	$\overline{P_J}$
Shanxi	0.787
Zhejiang	0.756
Guangdong	0.725
Hubei	0.722
Jilin	0.720
Anhui	0.661
Heilongjiang	0.576
Inner Mongolia	0.559

Level Three

Province	$\overline{P_J}$
Henan	0.498
Liaoning	0.489
Xinjiang	0.422
Gansu	0.419
Shanxi	0.384
Shandong	0.321

Level Four

Province	$\overline{P_J}$
Jiangsu	-0.394
Hebei	-0.455
Beijing	-0.525
Tianjin	-0.898

Level Five

Province	$\overline{P_J}$
Shanghai	-3.293
Ningxia	-6.556

The five levels divide China into five areas. *(Please see the Figure2 below.)* Our further discussion will base on the areas.

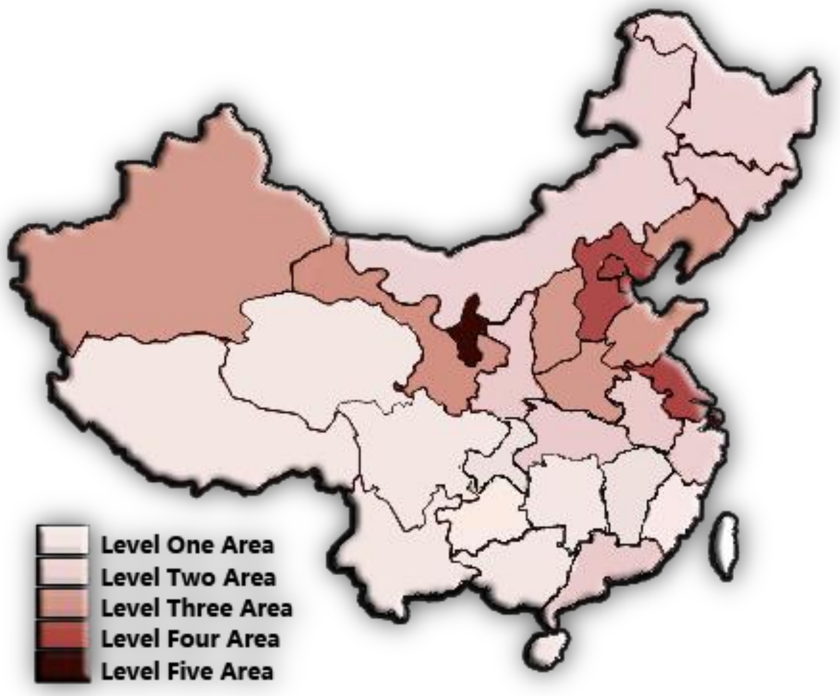

Figure2: The five areas of China

From the figure, we can clearly understand that Level one area is humid; Level two area is semi-humid, Level three area is neither too wet nor too dry; Level four area is arid and Level four area is the aridest area of China.

3.3 The Sine curve fitting model of freshwater supply

3.3.1 Basic analysis of freshwater supply

To predict China's freshwater supply of 2025 more accurately, we observe the data from 2004 to 2011 meticulously and find that the Sine curve is a better fitting model for water supply of every area， and the period is about two years. *(Please see the Figure3 below).*Also, we can account for this phenomenon with water cycle. Because of the water cycle, the supply of each area will fluctuate with the water movement.

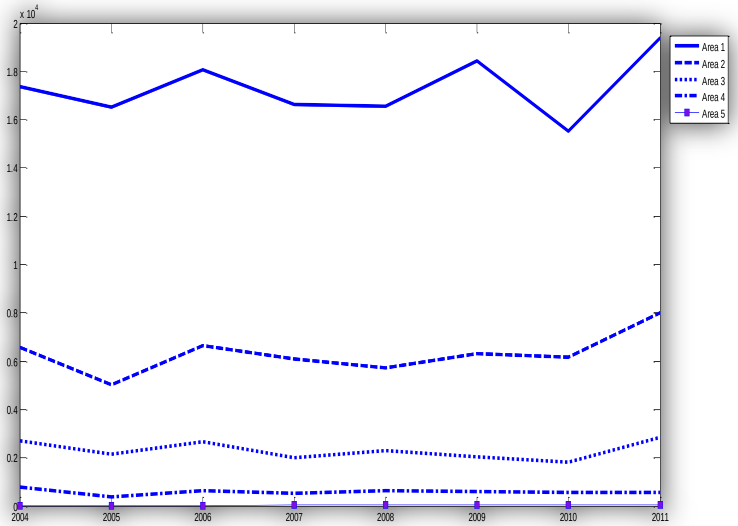

Figure3: The water supply of every area

3.3.3 The fundamentals of the problem of curve fitting

The problem of curve fitting is that we have known a set of two-dimensional data that are n points (x_i,y_i) in a plane (i =1, 2… n), x_i is different from each other, and find a function y=f(x) to let f(x) be closest to all data points, that is best curve fitting.

Linear least square method is the most commonly used method to solve curve fitting problem, and its basic idea is to let

$$f(x) = a_1 r_1(x) + a_2 r_2(x) + \cdots + a_m r_m(x)$$

Among that:

$r_k(x)$ are a set of linearly independent functions which are selected before;

a_k are undetermined coefficients(k=1,2,…,m; m<n).

Fitting criterion is to let the sum of squares of the distance between y_i(i = 1, 2… n) and $f(x_i)$be the least, which is also called the least squares criterion.

1. The determination of the coefficients a_k:

We write:

$$J(a_1, \dots, a_m) = \sum_{i=1}^{n} \delta_i^2 = \sum_{i=1}^{n} [f(x_i) - y_i]^2$$

In order to get $a_1, \dots, a_m$ which let J be the least, we only need the requirement of extreme value$\frac{\partial J}{\partial a_j} = 0$ (j=1… m) to obtain linear equations that about $a_1, \dots, a_m$:

$$\sum_{i=1}^{n} r_j(x_i) \left[\sum_{k=1}^{m} r_k(x_i) - y_i \right] = 0, j = 1 \dots m$$

Simplify the formula:

$$\sum_{k=1}^{m} a_k \left[\sum_{k=1}^{m} r_j(x_i) r_k(x_i)\right] = \sum_{k=1}^{m} r_j(x_i) y_i \,, j = 1 \dots m$$

When

$$R = \begin{bmatrix} r_1(x_1) \dots & r_m(x_1) \\ \dots & \dots \\ r_1(x_n) \dots & r_m(x_n) \end{bmatrix},$$

$$A=[a_1, \dots, a_m]^T,\ Y=[y_1, \dots, y_m]^T$$

Then we can get

$$R^T R A = R^T Y$$

When $\{r_1(x),\dots,\ r_m(x)\}$ is linearly independent, R is full column rank, so equations have unique solution:

$$A=R^T R^{-1} R^T Y$$

When we use linear least squares fit to the problem of curve fitting, the key step is to choose the most suitable $r_i(x_i)$.we can choose $r_i(x_i)$ by analyzing the mechanism or drawing the figure to judge what $r_i(x_i)$ to choose

Here, our group chooses the sine curve by drawing the figure to judge what $r_i(x_i)$ to choose.

3.3.3 Our Sine curve fitting model

To explain our model better, we take Level One area as an example. The statistic data of level one area from 2004 to 2011 is below: *(Please see the Table4 below)*

Table4: The freshwater supply of level one area

2004	2005	2006	2007	2008	2009	2010	2011
17348.52	16525.99	18050.96	16631.01	16538.13	18435	15532.18	19409.85

Building the fitting function:

$$y = a*(\sin(3*x - \frac{p_i}{6})) + c$$

where

a = a constant amount (the effect in the values of year on the values of supply water)

c= a constant amount (what one earns with zero education);

x= the values of the year;

y=the quantity of freshwater supply

Taking the data into the fitting function to calculate the parameters, we get can coefficients (with 95% confidence bounds):

a =769.3 (-466.9, 2005)

c = 1.728e+004 (1.628e+004, 1.828e+004)

And the fitting curve is *(Please see the Figure4 below)*:

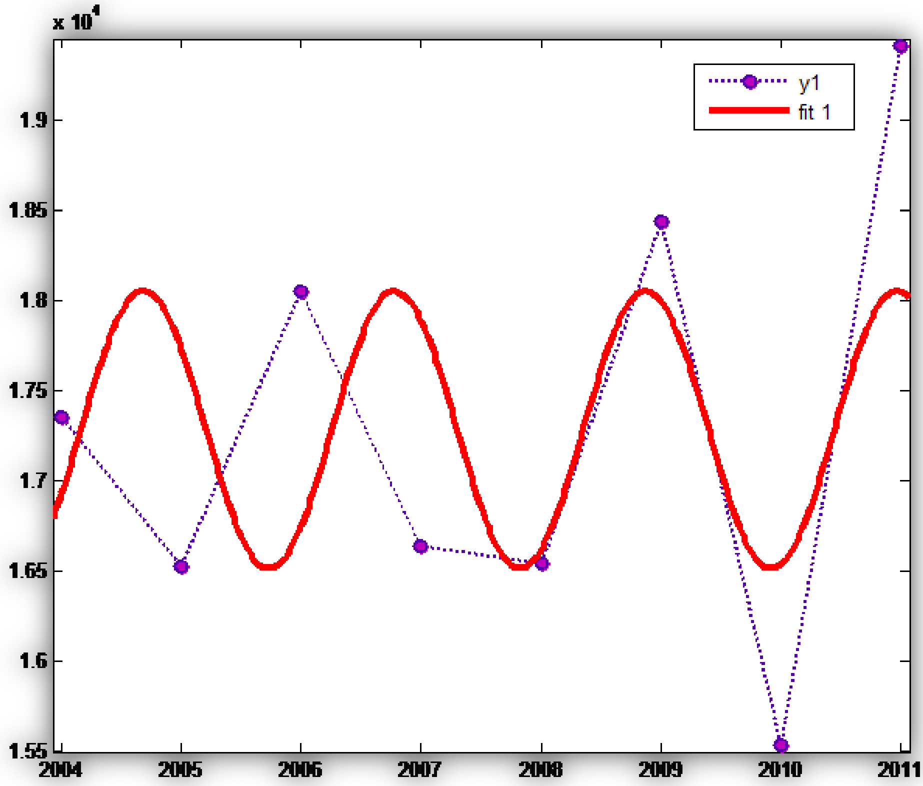

Figure4: The Sine fitting curve of 2004 to 2011

Then we will get freshwater prediction of 2025 is 17593 hundred Million Cubic Meters.

Similarly, we have applied the Sine curve fitting model to the other 4 areas and get the prediction of freshwater supply of 2025 as follow *(Please see the Table5 below)*

Table5: The freshwater supply of five areas of 2025

The area level number	**1**	**2**	**3**	**4**	**5**
Supply(100Million Cubic Meters)	17593	6433.31	2534.61	535.723	34.7198

3.4 Models of freshwater demand

3.4.1 Grey models (GM) of freshwater demand

3.4.1.1 Basic Analysis

We apply Grey model (GM) prediction [3] to the three aspects of water demand: industrial water, agricultural water and ecological water. As the demand of water for living is closed related to the population of China, we calculate it alone later.

3.4.1.2 The fundamentals of Grey model (GM)

GM (1, 1) represents gray model which is the first order differential equations and only have one variable.

1. GM(1,1) model prediction method

We have known reference data column $x^{(0)}=(x^{(0)}(1), x^{(0)}(2), \dots, x^{(0)}(n))$, one time accumulate generate order (1-AGO)

$$x^{(1)}=(x^{(1)}(1), x^{(1)}(2), \dots, x^{(0)}(n))$$
$$=(x^{(0)}(1), x^{(0)}(1)+x^{(0)}(2), \dots, x^{(0)}(1)+\cdots+x^{(0)}(n))$$

Among that:

$x^{(i)}(k)=\sum_{i=1}^{n} x^{(0)}(i)$, k=1, 2… n. The mean generating sequence $x^{(1)}$:

$$z^{(1)}=(z^{(1)}(2), z^{(1)}(3), \dots, z^{(1)}(n)),$$

Among that:

$$z^{(1)}(k)=0.5x^{(1)}(k)+0.5x^{(1)}(k-1), k=2,3,\dots n.$$

Establish gray differential equation:

$$x^{(0)}(k)+az^{(1)}(k)=b, k=2,3,\dots n,$$

Whose albino differential equation is:

$$\frac{dx^{(1)}}{dt}+ax^{(1)}(t)=b.$$

We write $\mathbf{u}=[a,b]^T$, $\mathbf{Y}=[x^{(0)}(2), x^{(0)}(3), \dots, x^{(0)}(n)]^T$, $\mathbf{B}=\begin{bmatrix} -z^{(1)}(2) & 1 \\ -z^{(1)}(3) & 1 \\ \vdots & \vdots \\ -z^{(1)}(N) & 1 \end{bmatrix}$, then

using the method of least squares, we obtain the estimated value of **u** which lets J(**u**)=(**Y**-**Bu**)(**Y**-**Bu**) be least is

$$\hat{\mathbf{u}}=[\hat{a},\hat{b}]^T=(\mathbf{B}^T\boldsymbol{B})^{-1}\mathbf{B}^T\boldsymbol{Y}$$

So we solve the equation and obtain:

$$\hat{x}^{(1)}(k+1)=(x^{(0)}(1)-\frac{\hat{b}}{\hat{a}})e^{-\hat{a}t}+\frac{\hat{b}}{\hat{a}}\ ,k=0,1,..,n-1,..$$

2. GM(1,1) model prediction steps

1).The testing and deal of data

First, in order to guarantee the feasibility of modeling method, we need to do necessary testing and deal for known data column. Assuming reference data is $x^{(0)}=(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n))$, we calculate the level ratio of sequence

$$\lambda(k)=\frac{x^{(0)}(k-1)}{x^{(0)}(k)}, \text{k=2, 3…n.}$$

If all level ratios $\lambda(k)$ are in capacity coverage $\Theta=(e^{-\frac{2}{n+1}}, e^{\frac{2}{n+1}})$, then sequence $x^{(0)}$ can be as the data of the model GM (1, 1) to do gray prediction. Otherwise, we need to do necessary converting deal for sequence $x^{(0)}$ to let them be in the capacity coverage. Choosing the constant c, do the translating:

$$y^{(0)}(k)=x^{(0)}(k)+c, k=1,2,\ldots,n-1$$

And make the radio of sequence $y^{(0)}=(y^{(0)}(1), y^{(0)}(2), \ldots, y^{(0)}(n))$:

$$\lambda_y(k)=\frac{y^{(0)}(k-1)}{y(k)}\in\Theta, \text{k=2, 3…n.}$$

2) Modeling

Establish GM(1,1) model, and then we can obtain predictive value

$$\hat{x}^{(1)}(k+1)=\left(x^{(0)}(1)-\frac{\hat{b}}{\hat{a}}\right)e^{-\hat{a}t}+\frac{\hat{b}}{\hat{a}}, k=0,1,\ldots,n-1,\ldots,$$

And $\hat{x}^{(0)}(k+1)=\hat{x}^{(1)}(k+1)-\hat{x}^{(1)}(k), k=1,2,\ldots,n-1,\ldots$

3) Testing predictive value

Residuals test:

Let relative errors be $\varepsilon(k)$, and then calculate

$$\varepsilon(k)=\frac{x^{(0)}(k)-\hat{x}^{(0)}(k)}{x^{(0)}(k)}, \text{k=1, 2,…,n}$$

Here, $\hat{x}^{(0)}(1)=x^{(0)}(1)$, if $\varepsilon(k)<0.2$, we can believe that it achieves the general requirements; if $\varepsilon(k)<0.1$, we can believe that it achieves the higher requirements.

Testing deviation value of level ratio:

First, we calculate level ratio $\varepsilon(k)$ by reference data, and then obtain corresponding deviation value of level ratio through development coefficient a:

$$\rho(k)=1-\left(\frac{1-0.5a}{1+0.5a}\right)\lambda(k)$$

If $\rho(k)<0.2$, we can believe that it achieves the general requirements; if $\rho(k)<0.1$, we can believe that it achieves the higher requirements.

4)Forecasting

We obtain the predictive value in specified area through GM(1,1), and according to the need of practical problems, we give the corresponding forecast .

3.4.1.3 Solutions

The estimated freshwater demand of industrial of the five level areas is as follows *(We choose two areas as examples to show you because there are too many Figures. Please see the Figure5 below)*

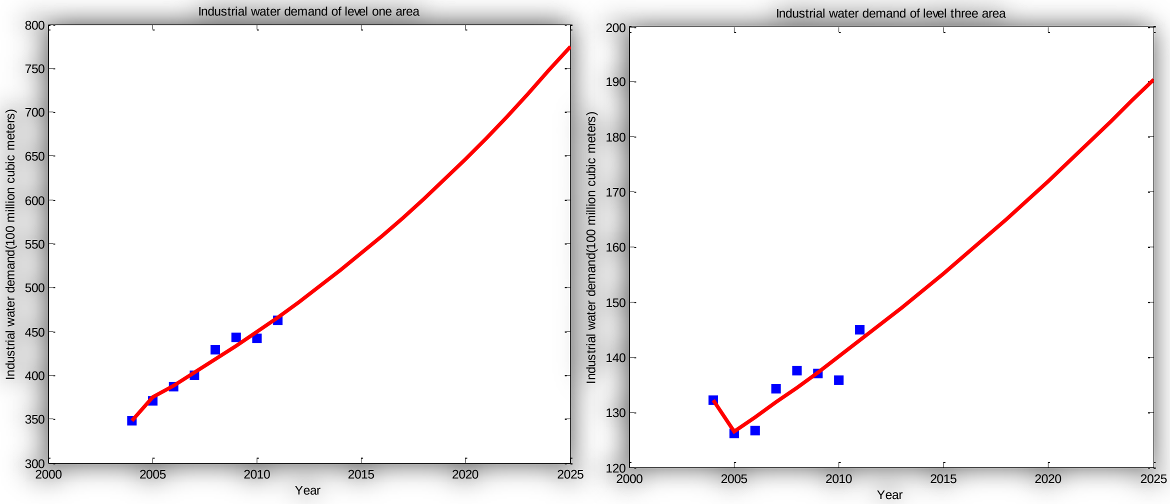

Figure5: The industrial water demand of level one and three areas

The estimated freshwater demand of agricultural of the five level areas is as follows *(We choose two areas as examples to show you because there are too many figures. Please see the Figure6 below)*

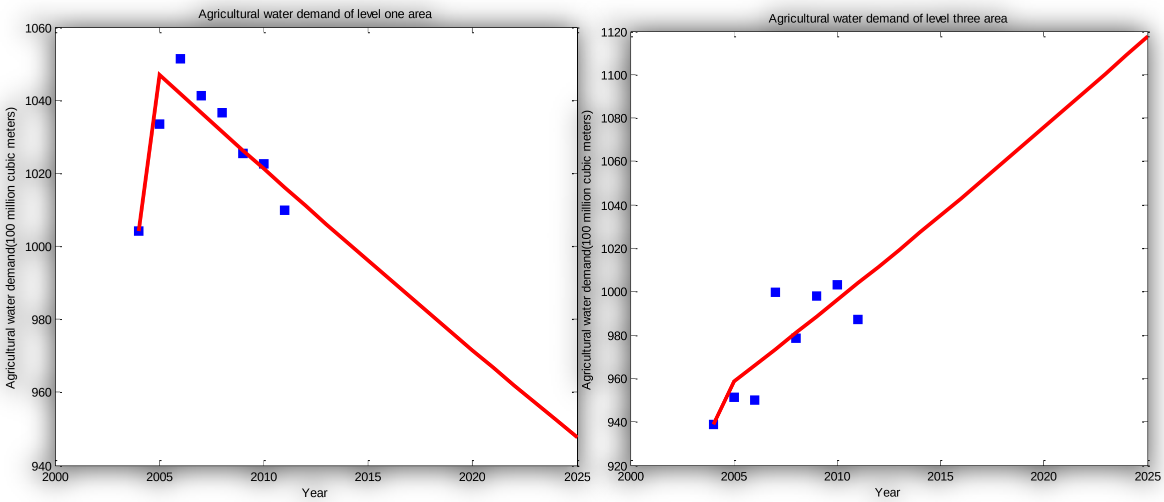

Figure6: The agricultural water demand of level one and three areas

The estimated freshwater demand of ecological of the five level areas is as follows *(We choose two areas as examples to show you because there are too many figures. Please see the Figure7 below)*

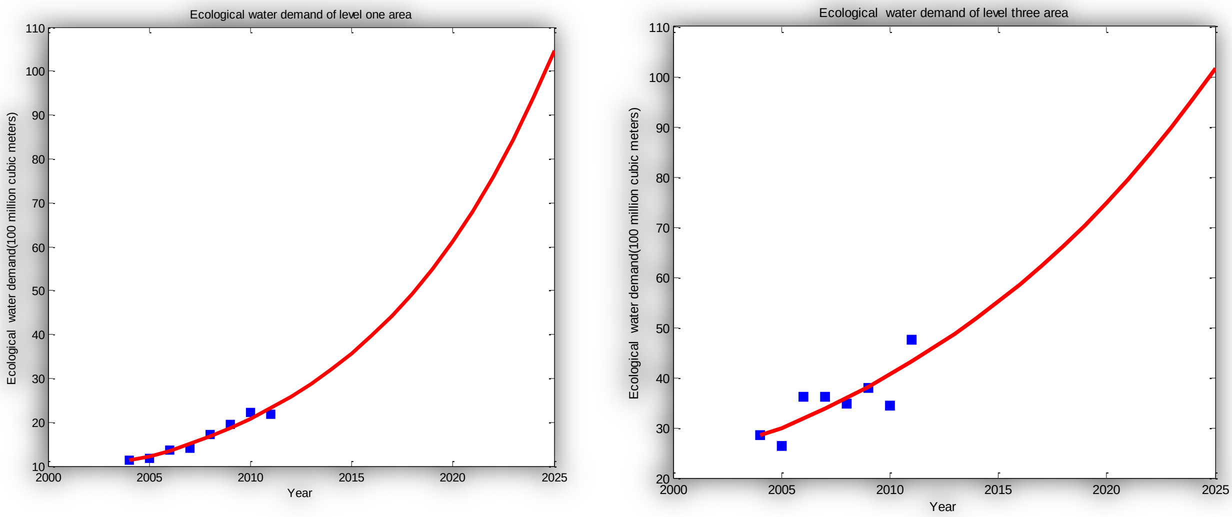

Figure7: The ecological water demand of level one and three areas

The table below wholly shows you the demand of water of 2025 we predict above **(***Please see the Table6 below)*

Table6: the three aspects of freshwater demand of five areas of 2025

Demand(100Million Cubic Meters)	**1**	**2**	**3**	**4**	**5**
Industrial water	947.7	1423.6	1117.6	514.097	64.428
Agricultural water	774.7451	808.5614	190.4713	220.4778	105.9203
Ecological water	104.7508	86.7158	101.5851	10.6855	2.6679

3.4.2 The Logistic model and Logarithmic curve fitting model for calculating the water for living

3.4.2.1 Basic Analysis

The demand of water for living is closed related to the population of China, so when we forecast the demand of living water, we couldn't use the living water data from formal years only. We must take the changes of population into account.

On the one hand, we use Logistic population model [2] calculate the population size of the year 2025, on the other hand we use GM to calculate the rate of population of every areas. Then we use the demand of living water per person multiplied the population, we get the total amount of living water.

3.4.2.2 The Logistic model

We assume that:
N: The total population of China
K: The Limit population, with its resources
t_i: The year we use and predict
r: The natural population growth rate, when the population isn't to large

Because K is the limit population and r is the natural population growth rate, we define the relative growth rate of population $r(1-\frac{N}{K})$ is more reasonable. So N(t) is satisfied with the equation:

$$\begin{cases} \frac{dN}{dt} = r\left(1-\frac{N}{K}\right)N \\ N(t_0) = N_0 \end{cases},$$

Calculating we get：

$$N(t) = \frac{N_0 K e^{r(t-t_0)}}{K + N_0(e^{r(t-t_0)} - 1)} = \frac{K}{1 + (\frac{K}{N_0} - 1)e^{-r(t-t_0)}}$$

When

N_0=1.26743× 10^9, t_0 =2000

And the population of formal years is below **(***Please see the Table6 below):*

Table7: the population of China (2000-2011)

Year	2000	2001	2002	2003	2004	2005
Population（10^5）	126743	127627	128453	129227	129988	130756
Year	**2006**	**2007**	**2008**	**2009**	**2010**	**2011**
Population（10^5）	131448	132129	132802	133450	134091	134735

We can get:

K = 6.4073×$10^{9;}$ r =0.0058
So according to the Logistic model, we can get the population of 2025 is:

$$N = 1.4649 \times 10^9$$

3.4.2.3 GM for calculating the rate of population of each area

Similarly, we use the grey model to calculate the rate of population of each area and get the result **(***Please see the Figure8 and Table 8 below).*

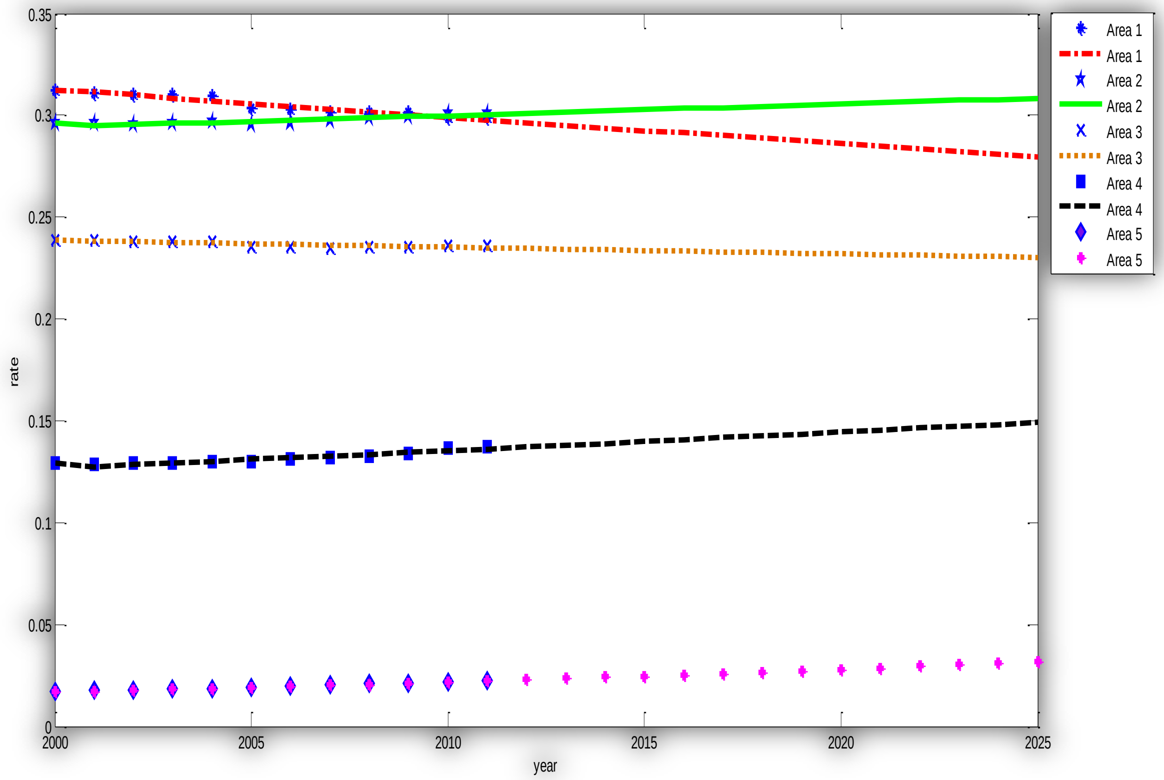

Figure8: The ecological water demand of level one and three areas

Table8: the population rate of each area

The area number	**1**	**2**	**3**	**4**	**5**
The rate	0.2798	0.3085	0.2303	0.1492	0.031
Population(10 thousand person)	40988	45192	33737	21856	4658

3.4.2.4 The Logarithmic curve fitting model

We use the Logarithmic curve to match the average domestic water-consumption per person, which also is curve fitting similarly to the Sine curve fitting.

Building the fitting function:

$$y = a*\log(x+t)+b$$

where

a = a constant amount

b= a constant amount (what one earns with zero education);

t = a constant amount

x= the values of the year;

y=the quantity of the average domestic water-consumption per person

Taking the data into the fitting function to calculate the parameters, we get can coefficients (with 95% confidence bounds): *(Please see the Table9 below)*:

Table9: Coefficients

	1	2	3	4	5
a	0.3468	0.2148	0.1301	0.1578	0.04988
b	-2.634	-1.628	-0.985	-1.194	-0.3712
t	10.71	10.71	-9.029	-9.409	10.04

And the fitting curve is *(Please see the Figure9 below, we take two and four level area as an example)*:

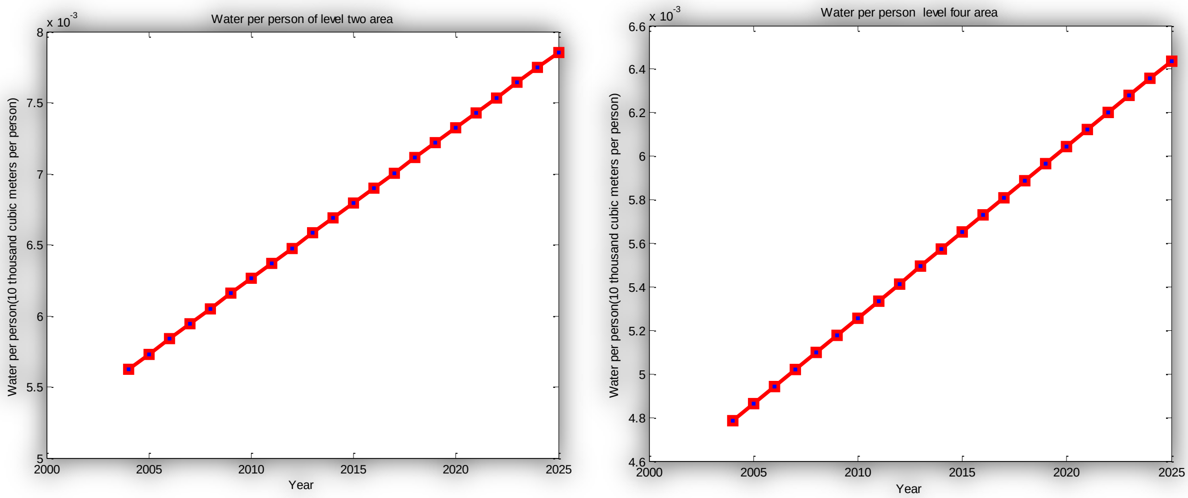

Figure9: The average domestic water-consumption per person

Table10: The average domestic water-consumption per person

The area number	1	2	3	4	5
water-consumption per person(cubic meter per person)	87.3718	78.5213	50.2974	64.3542	88.2537

3.5 The situation of supply and demand of 2025

Up to now, we can obtain the freshwater supply and demand of each area of 2025, and calculate the differences between them. *(Please see the Table11 below)*

Table11: The situation of supply and demand of 2025

	1	2	3	4	5
Supply	17593	6433.31	2534.61	535.723	34.7198
Living demand	358.1187	354.8548	169.6866	140.6545	41.11194
Industrial demand	774.7451	808.5614	190.4713	220.4778	105.9203
Agricultural demand	947.7	1423.6	1117.6	514.097	64.428
Ecological demand	104.7508	86.7158	101.5851	10.6855	2.6679
Total demand	2185.315	2673.732	1579.343	885.9148	214.1281
Differences	15407.69	3759.578	955.267	-350.192	-179.408

All the amounts above are measured by 100Million Cubic Meters.

From the table, we can obviously find that the level four and five areas are still having the water shortages when time comes to 2025.

So we have to transfer water from level one and two area where the freshwater is abundant or desalinate water from the sea.

3.6 Modeling testing

We take the area one as an example to show the results of testing. And the others are similar.

Table12: The GM(1,1) Testing

year	primitive value	predicted value	Residuals	Relative error	Ratio error
2004	347.5495	347.5495	0	0	
2005	370.47	374.5847	-4.1147	0.0111	0.0272
2006	386.43	388.4458	-2.0158	0.0052	0.0058
2007	399.5507	402.8199	-3.2692	0.0082	-0.003
2008	428.6921	417.7259	10.9663	0.0256	0.0335
2009	442.75	433.1834	9.5666	0.0216	-0.0041
2010	441.8676	449.2129	-7.3453	0.0166	-0.0391
2011	462.2595	465.8356	-3.5761	0.0077	0.0087

Upon examination, the accuracy of the model is high. We can use the model to predict.

4. Storage and movement, de-salinization, and conservation

4.1 Minimum Spanning Tree (MTS) algorithm of movement

Up to now, we can obtain water-scarce regions, Level Four Area and Level Five Area. The water shortage in the total is up to 52.96 billion cubic meters in the year 2025. We picked the most water-shortage cities from the areas, and the cities are Ningxia Guyuan, Beijing, Tianjin, Shijiazhuang, Shanghai and Jiangsu Xuzhou. To solve the problem of water shortage in these two areas, we select seven cities from Level One Area and Level Two Area whose water is abundant to provide, and the cities are Lasa, Chengdu, Wuhan, Kunming, Guiyang, Fuzhou and Changsha. We can see the above-mentioned cities in the following figure *(Please see the Figure10 below)*:

Figure10 Water-shortage cities and water-abundant cities

In this figure, we mark these water-shortage cities in red and cities that provide water in blue. Through connecting these cities, we can get a network graph which we can see in the following figure *(Please see the Figure11 below)*:

Figure11 The network graph of cities

Then, we mark the weights of all edges, which are the distances between cities actually. There, in order to obtain the smallest cost, we use the Minimum Spanning Tree algorithm. We choose the Prim algorithm for the calculating of MST.

Prim algorithm:

Let G= (V, E) be a connected weighted graph, V = {1, 2… n}, firstly, set S = {1}, then S is a proper subset of V, and do following greedy select: Select the vertex j which satisfies the condition i ϵ S, j ϵ V - S and c [i] [j] is the smallest edge, and add j to the S.This process continues until S = V.All edges selected in this process constitute a the smallest spanning tree of G exactly .

And the tree we got is in the following figure *(Please see the Figure12 below)*:

Figure12 The Minimum Spanning Tree we get

Next, we calculate the sum of the weights of all edges in this tree, which is also the total distance of route, and we get 6195.8 km.

To calculate the cost of this model, we reference the South-to-North Water Diversion Project [4].

And we define the following parameters:

a: The length of the route of South-to-North Water Diversion Project

b: The amount of transferred water of South-to-North Water Diversion Project

c: The total cost of South-to-North Water Diversion Project

d: The length of route of our model

e: The amount of water transferred in our model

f: The total cost of our model

g: The cost of water transferring per cubic meter per kilometer

Then we build a cost function:

$$g = \frac{c}{a*b}$$

$$f = g*d*e$$

By referencing data, we know that the total length of South-to-North Water Diversion Project is 4350 kilometer, and the cost is 500 billion RMB, and the amount of water transferred is 44.8 billion cubic meter. And through our calculation and prediction, we get that we need transfer 52.96 billion cubic meter water.

Using this function to calculate, we can get the cost of our model, which is 841.88 billion RMB. Furthermore, we can calculate the cost of water transferring per cubic meter which is 15.896 RMB.

4.2 The storage analysis based one the Fermat point

4.2.1 Basic analysis of where to build the reservoir

We choose Beijing，Tianjin and Shi Jiazhuang as the representative cities to discuss the problem of storage. *(Please see the Figure13 below)*:

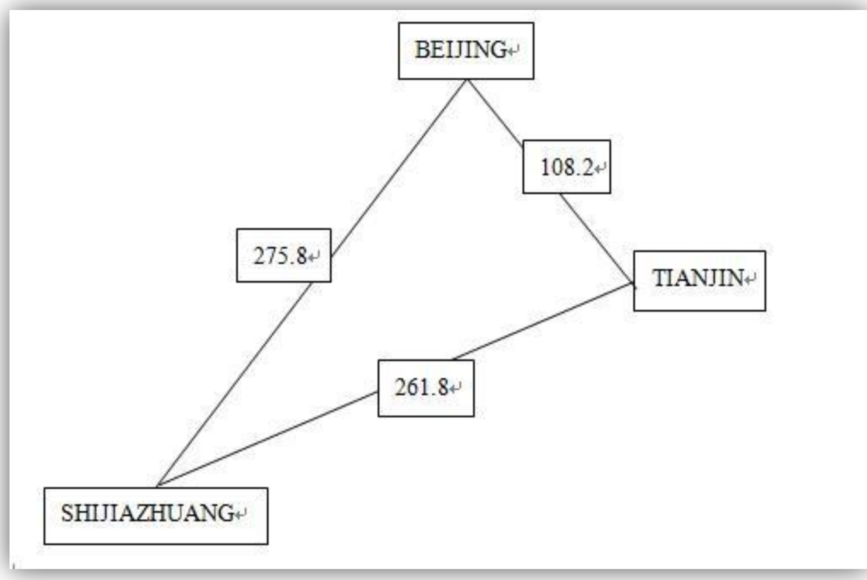

Figure13 The three cities we choose

Apparently they are geographically close to and belong to the same area where is serious of water shortage. So we decide to build an artificial reservoir to supply water for the three cities.

We assume that we can build a reservoir in any place without thinking any other factors. So we need to choose a place to the three cities with the minimum distance in order to make the water transportation pipeline cost least. We can conclude the point is a Fermat point.

4.2.2 The way to find the Fermat point

In geometry the Fermat point of a triangle, also called Torricelli point, is a point such that the total distance from the three vertices of the triangle to the point is the minimum possible.

In order to locate the Fermat point of a triangle with largest angle at most 120°:

1. Construct two equilateral triangles on any of the three sides of the given triangle.
2. For each new vertex of the equilateral triangle, draw a line from it to the opposite triangle's vertex.
3. The two lines intersect at the Fermat point.

An alternate method is the following:

1. On any two of the three sides, construct two isosceles triangles, with base the side in question, 30-degree angles at the base, and vertices lying outside the original triangle.
2. Draw two circles, each with a center on the vertex of the just constructed isosceles triangles and radius the identical side of the isosceles triangles.
3. The intersection inside the original triangle between the two circles is the Fermat point.

When a triangle has an angle greater than 120°, the Fermat point is sited at the obtuse-angled vertex. *(Please see the Figure14 below)*:

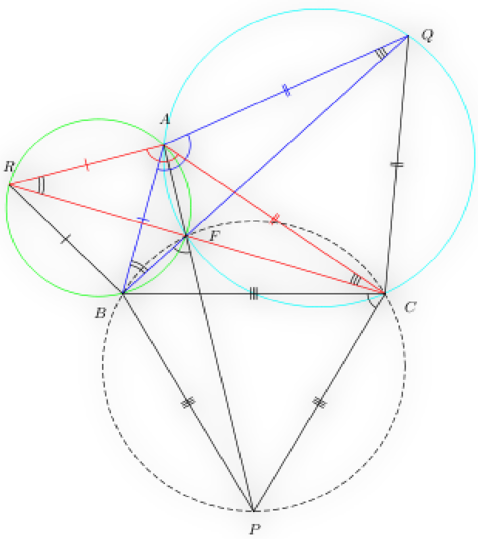

Figure14 The way to get the Fermat point

4.2.3 The application of Fermat point in our model

In our model, we establish a Cartesian coordinate system.

A is Tianjin, B is Shi Jiazhuang, C is Beijing. *(Please see the Figure15 below)*:

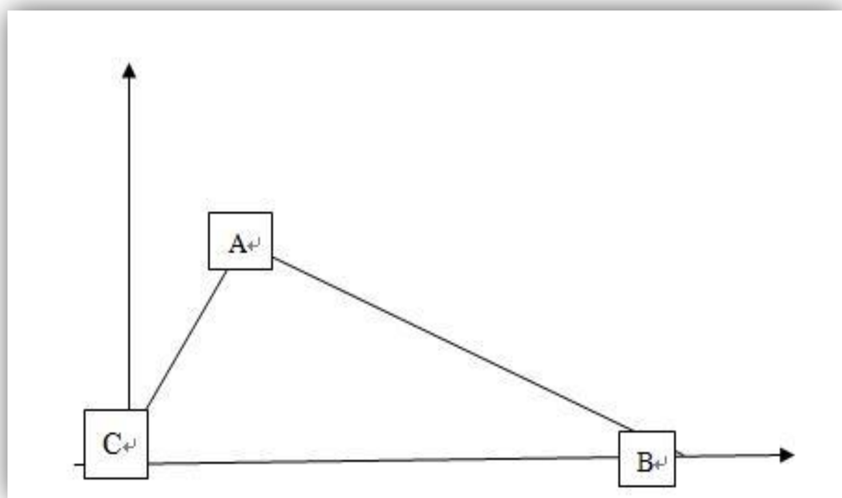

Figure15 Three cities in the coordinate system

According to the cosine theorem:

Cos ACB = (AC^2+BC^2-AB^2)/ (2*AC*BC)

In our model, AC=108.2, BC=275.8, AC=261.8, the coordinates of A is (AC* cos ACB, AC*sin ACB).

So, the result is :

A (0, 0), B (275.8, 0), C (34.8688, 102.4276)

Then we use Matlab to get the Fermat point: (50, 53)

So, we should build the reservoir in this place.

4.2.4 The cost t of building a reservoir

According to the plan of the reservoir in Chongqing City in 2012, we know that building 10 medium-sized reservoirs need a total investment of 4 billion RMB after the completion of new capacity of 0.124 billion cubic meters.[5] It can provide 0.18 billion cubic meters of water every year.

We define the following parameters:

Q: The amount of water needed

Q1: The reservoir volume

V: The reservoir investment

From the data above, we can obtain the function:

Q1=Q*(1.24/1.8)

V=Q1*(40/1.24)

So we can get the function as follows,

V=Q1*40/1.24

=Q*40/1.8

=22.2222*Q

Next, we roughly calculate the volume of the reservoir.

Here we approximate think that the ratio of the water needed of Jiangsu Province remains unchanged of the level forth area.

We can find out the ratio is：17.98%

In 2025, the water shortage of level four area is 35.01918 billion cubic meter, so Beijing, Tianjin and Hebei water shortage is:

$$Q=350.1918*17.98\%$$
$$=6.29578 \text{ billion cubic meter}$$

Then the total investment is：

$$V=22.2222*Q$$
$$=139.91 \text{ billion}$$

4.3 De-salinization

Sea water desalination is an open source incremental technology to realize utilization of water resources, and it uses sea water to produce fresh water. In China, desalination is an important measure to alleviate the water shortage situation. To consider the cost of Sea water desalination, we research by reference Tianjin, whose productivity accounting for one-third of the national total amount.

Sea water cannot be used directly because its salt content is very high. We use mainly two methods to desalination, including distillation and reverse osmosis method. Application surface of reverse osmosis method is very wide, and the desalination rate is very high, so it is widely used. Therefore, we mainly discuss reverse osmosis method [6].

To calculate the cost of reverse osmosis method, we reference the Tianjin standard demonstration project.

And we define the following parameters (which represents the cost of producing one cubic meter):

a: The cost of construction
b: The cost of electricity
c: The cost of remedy
d: The cost of membrane replacement
e: The cost of service
f: The cost of staff salaries
g: The total cost

Then we build a cost function:

$$g = a + b + c + d + e + f$$

By referencing data [7], we know that desalination plant construction costs 7.5 million, and age limit is 15 years, and its water production per day is 1000 cubic meter, and it has 4 workers whose average annual salary is up to 20 thousand RMB. The price of electricity in Tianjin is 0.65RMB per kWh, and power consumption for producing cubic meter water is 4 kWh.

The cost of remedy is 0.45 RMB per cubic meter, and cost of membrane replacement is 0.89 RMB. The annual maintenance fee is 2% of the total investment in equipment.

Through the data above, we can get that the amount of water is 1000*365 cubic meter ,the cost of construction per cubic meter is 7500000 / 15 / (1000*365), the cost of staff salaries is 4*20000/(1000*365), and the cost of service is

$$\frac{7500000 * 2\%}{(1000 * 365)}$$

So we can obtain that the cost is 5.94 RMB per cubic meter.
Further, we can calculate the percentage of each cost. *(Please see the Table13 below)*:

Table13 The percentage of each cost

electricity	staff salary	remedy	membrane replacement	service	construction
43.77%	3.69%	7.58%	14.98%	6.92%	23.06%

So we can see that electricity costs account for 43.77%, investment costs account for 23.06%, and the two together accounted for 66.83%.Therefore, to reduce the cost of the fresh water, the key is to reduce the project investment and energy consumption.

4.4 Conservation

We analyzed the 2004-2010 sewage governance and investment [8].
Model: We define P to describe the change of an array.

$$P_i = \frac{x_i - \bar{x}}{\bar{x}}$$

$$\bar{x} = \frac{\sum_{i=1}^{n} x_i}{n}$$

We promise that if all P_i satisfy $|P_i| < 0.05$, then we can draw a conclusion that the data do not have Significant changes.
In this way, we find that the data in this period almost remain the same.
For example, we can see the compliance rate below:

Table14 The compliance rate

Compliance rate	P
0.906962705	-0.0176
0.911963247	-0.01218
0.906956693	-0.01761
0.91657212	-0.00719
0.924467548	0.001362
0.942354266	0.020736
0.953196824	0.03248

Their average is below:
Unit (Ten thousand cubic meters)
Total volume of sewage: 2378012.138
Qualified: 2233986
Unqualified: 182545.0921

Investment in governance sewage (Ten thousand Yuan): 1514429.114

Compliance rate: 0.92321%

Conclusion:

We can see that the sewage treatment is very good in China.

Only handling the industrial wastewater to meet emission standards, can we protect more water from polluting. Besides, we should plant more trees and save water.

5. The economic physical and environmental implications

Water diversion project is a sparkling point of China's irrigation works in the 21 st century which has significant social, economic and ecological benefits. Desalination is a great works, too.

First, solve the problem of shortage of water resources in the northern region, and it promotes this region's economic, social development, many people can solve the long-term consumption of high fluoride water and brackish water. Because it is a hard but big work, it solves the employment problem of many people. The phenomenon of over-exploitation of groundwater is effectively alleviating .Besides, it can increase ecological and agricultural water supply, deterioration of aquatic ecosystems of the northern region, and gradually restore and improve the ecological environment.

Where the diversion route passes sometimes need large-scale migration. It affects the survival of some animals because this project changes their original living environment. Besides, it costs a large quantity of human, material and financial resources.

6. Strength and Weakness

6.1 Strength

- The idea of partition. When predicting the water situations, we propose the level of water shortage, and classify the province group by situations.
- We take into account the impact of the water cycle in the prediction of the total water resources, and we use trigonometric fitting because of cyclical changes in water, and set the corresponding period.
- When we predict the water consumption of each area, water consumption is divided into several parts, and is predicted respectively, and we take into account the growth of the population in the prediction of domestic water.
- When we select the diversion line, we use the idea of minimum spanning tree, so

that the total length of diversion route is the shortest, and the cost reduce.

- We use Fermat point to select the reservoir addresses, shortening the water supply path length between three points, so we reduce the cost.

6.2 Weakness

- When we select the water diversion route, we only choose the length of the path as weight, without taking into account the topography, climate and other factors.
- In the desalination, because we could not find the data of tariff and staff salaries every year, we cannot predict the data of year 2025, and then we only using the current data to calculate the cost of desalination.

7. References

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[8]The 2004-2010 sewage governance and investment, http://www.stats.gov.cn/tjsj/ndsj/

8. The position paper

Dear Sir/Madam:

According to our work, we draw a conclusion that water shortage situation of China is grim. In order to meet the needs of water supply in 2025, the measures below must be taken:

First, we should take a low-cost and efficient water diversion scheme to cater for the need of some cities where people can't drink fresh water.

Second, control the price of electricity appropriately to encourage desalination of sea water in coastal cities because it has great influence on desalination costs. By our research, we obtain that the cost of the diversion is higher than desalination, so we think that coastal cities should priority to get fresh water through desalination to meet their need. What's more, through study of the cost of desalination we obtain that electricity costs account for 43.77%, investment costs account for 23.06%, and the two together accounted for 66.83%.Therefore, to reduce the cost of the fresh water, the key is to reduce the project investment and energy consumption.

Third, build some artificial reservoirs properly to increase the water storage capacity of some area short of water.

Last but not the least, enhance wastewater treatment compliance rate and protect the water from being polluted.

Why our model is credible?

Our model is obtained through the analysis of large amounts of data, and it is established after comparing, optimized and improved. Most important, it is in line with China's national conditions and the actual situation

If all of my suggestions are taken, I promise China will easy to deal with the crisis of water resources in 2025. You are shouldering an important mission.

Yours sincerely,
Team #23178